THEORY AND BEHAVIOR OF SINGLE OBJECT AUCTIONS

James C. Cox, Bruce Roberson and Vernon L. Smith

I. INTRODUCTION

This paper addresses the subject of the relation between the predictions of economic theory and bidder behavior in the Dutch auction, the English auction, the first-price sealed-bid auction, and the second-price sealed-bid auction of a single item. The four types of auction market are defined as follows.

1. *Dutch:* In this auction the offer price starts at an amount believed to be higher than any bidder is willing to pay and is lowered by an auctioneer or a clock device until one of the bidders accepts the last price offer (Cassady, 1967, p. 67). The first and only bid is the sales price in the Dutch auction.

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2. *English:* This is the ". . . progressive auction, in which bids are freely made and announced until no purchaser wishes to make any further higher bid" (Vickrey, 1961, p. 14). The last bid is the sales price in the English auction.
3. *First-price:* This auction corresponds to ". . . the usual practice of calling for the tender of bids on the understanding that the highest . . . bid . . . will be accepted and executed in accordance with its own terms" (Vickrey, 1961, p. 20). In the first-price auction, the auctioned object is awarded to the highest bidder at a (sales) price equal to his bid.
4. *Second-price:* Under this auction procedure, bids are tendered on the understanding that the item will be awarded to the highest bidder, but at a price equal to the second highest bid (Vickrey, 1961, pp. 20–21). Cassady's (1967, pp. 152–153) description of how "book bids" are handled in the London stamp auction corresponds to the second-price auction.[1]

The Dutch and English auctions are commonly referred to as "oral" auctions, as distinct from the "written-bid" (i.e., sealed-bid) auctions. In fact, the feature that distinguishes the Dutch and English auctions from the sealed-bid auctions is the "real-time" element of the former auctions, not that bids are actually made orally. Thus, during the conduct of the Dutch and English auctions a bidder is able to observe some bidding behavior of his rivals. In the absence of collusion in a sealed-bid auction, a bidder is not able to make any observations of his rivals' bidding behavior.

In developing the economic theory of these four auction markets, we will be concerned with the implications of the expected utility hypothesis and the Nash equilibrium condition. The theory and the experimental design in this paper apply to the case where each bidder knows with certainty the monetary value that he places on the auctioned object but does not know the values that his rivals place on the auctioned object.

II. IMPLICATIONS OF THE EXPECTED UTILITY HYPOTHESIS

We understand the expected utility hypothesis to be the assumption that a bidder chooses his bid as if his objective were to maximize his expected (von Neuman–Morgenstern) utility of the money income gained from participating in an auction. In the present section of the paper, we explore the implications of the expected utility hypothesis for bidding behavior.

Define N as the number of bidders participating in the auction. Let the strictly increasing concave function u_i, $i = 1, 2, \ldots, N$, be the utility function for money income for the ith bidder. Adopt the normalization that $u_i(0)$ equals 0 for all i and that the utility of not bidding equals 0 for all bidders. Further assume that there is no utility or disutility associated with participating in the auction other than the utility of the monetary gain from the auction. The monetary value to bidder i of the auctioned object is denoted by v_i. Assume that v_i is known with certainty by bidder i and that v_i is positive for each i. The bid of the ith bidder is denoted by b_i and the auction market rules require b_i to be nonnegative.

Now consider the first-price sealed-bid auction. If the ith bidder submits the highest bid, then he obtains the money income $(v_i - b_i)$. If he does not submit the highest bid, then his monetary gain from participating in the auction is zero. Let $F_i(b_i)$ be the ith bidder's subjective probability that he can win the auction with a bid in the amount b_i. Thus the expected utility to bidder i of a bid in the amount b_i in the first-price auction is

$$U_i(b_i) = F_i(b_i)u_i(v_i - b_i). \tag{2.1}$$

Assume now, for simplicity, that the amount bid is a continuous variable and that the interval $[\underline{X}_i, \overline{X}_i]$ is the support of the probability distribution function F_i. Further assume that the expected utility function (2.1) is pseudoconcave and that there exists a unique positive expected utility-maximizing bid b_i^o. Then b_i^o will satisfy the following first-order condition:

$$0 = U_i'(b_i^o) = F_i'(b_i^o)u_i(v_i - b_i^o) - F_i(b_i^o)u_i'(v_i - b_i^o). \tag{2.2}$$

Also assume that b_i^o satisfies the second-order condition for a maximum,

$$\begin{aligned} 0 > U_i''(b_i^o) = {} & F_i''(b_i^o)u_i(v_i - b_i^o) \\ & - 2F_i'(b_i^o)u_i'(v_i - b_i^o) \\ & + F_i(b_i^o)u_i''(v_i - b_i^o). \end{aligned} \tag{2.3}$$

If $U_i''(\cdot)$ is negative on an interval subset of the domain of $U_i(\cdot)$, then statements (2.2) and (2.3) and the implicit function theorem imply that there exists a differentiable function ψ_i such that

$$\begin{aligned} b_i^o &= \psi_i(v_i) \\ &= v_i - u_i^{-1}(u_i'(v_i - b_i^o)\, F_i(b_i^o)/F_i'(b_i^o)) \end{aligned} \tag{2.4}$$

and

$$\psi_i'(v_i) = [F_i(b_i^o)u_i''(v_i - b_i^o) - F_i'(b_i^o)u_i'(v_i - b_i^o)]/U_i''(b_i^o), \tag{2.5}$$

where u_i^{-1} is the inverse of the utility of money income function. The function ψ_i is called the ith bidder's strategy function or his bid function.

The implications of the expected utility hypothesis for bidding behavior in the first-price sealed-bid auction follow immediately from statement

(2.4). First, the expected utility-maximizing bid is less than the value of the auctioned object. Therefore, the first-price sealed-bid auction is not a demand-revealing allocation mechanism. Secondly, the amount by which the object value exceeds the optimal bid depends on u_i and F_i; that is, it depends on the bidder's risk preferences and expectations about his rivals' bids. Since risk preferences and expectations can differ over individual bidders, the highest bid will not necessarily be submitted by the bidder who places the highest value on the auctioned object. Thus the first-price auction does not, in general, yield Pareto-efficient allocations. However, we can identify a set of conditions under which this auction would yield efficient allocations. Concavity and monotonicity of u_i and statements (2.2), (2.3), and (2.5) imply that the bid function ψ_i is increasing. If we now assume that all N bidders have the same risk preferences and expectations about their rivals' bids, then they will all have the same increasing bid function. In that special case, the first-price sealed-bid auction will yield Pareto-efficient allocations. However, if all bidders are not identical, then the conclusion that the first-price auction is efficient has been shown by the discussion in this paragraph to be untenable. We will consider this point again in Section III below.

Consider next the second-price sealed-bid auction. In that auction the highest bid is the winning bid but the price paid for the auctioned object by the winning bidder is the amount of the second highest bid. In the absence of perfect collusion among the bidders, an individual bidder will not know with certainty the bids of his rivals before he must decide on the amount of his own bid. Thus let the random variable y be the highest bid of any of the rivals of a particular bidder i. Let the ith bidder's expectations about y be represented by the cumulative distribution function G_i, with support $[\underline{Y}_i, \overline{Y}_i]$. If the ith bidder submits the highest bid, then he obtains the money income $(v_i - y)$. If he does not submit the highest bid, then his monetary gain from participating in the auction is zero. Thus the expected utility to bidder i of a bid in the amount $b_i \in [\underline{Y}_i, \overline{Y}_i]$ in the second-price auction is

$$V_i(b_i) = \int_{\underline{Y}_i}^{b} u_i(v_i - y)\, dG_i(y). \tag{2.6}$$

Consider the interesting case where $v_i \in [\underline{Y}_i, \overline{Y}_i]$ and, for simplicity, assume that the expected utility-maximizing bid of bidder i, b_i^*, satisfies the following first-order condition:

$$0 = V_i'(b_i^*) = u_i(v_i - b_i^*)G_i'(b_i^*). \tag{2.7}$$

Statement (2.7) and the normalization $u_i(0) = 0$ imply

$$b_i^* = v_i. \tag{2.8}$$

Thus the bidder's strategy function in the second-price auction is the identity map.

The implications of the expected utility hypothesis for bidding behavior in the second-price sealed-bid auction follow immediately from statement (2.8). The expected utility-maximizing bid is equal to the value of the auctioned object. Furthermore, it does not depend on the bidder's risk preferences or his expectations about his rivals' bids. Therefore, the outcome where each of the N bidders submits a bid equal to his object value is a dominant strategy equilibrium in the second-price auction. Thus the second-price auction is a demand-revealing allocation mechanism that will, in general, yield Pareto-efficient allocations. Furthermore, the winning bid will equal the largest of the values v_i, and the sales price will equal the second highest of those values.

An additional question of interest for comparison of alternative auction markets is the effect of auction market structure on the distribution of seller's revenue. The question is often posed in terms of a comparison across auction markets of the first two moments of the probability distribution of seller's revenue. Thus it is a matter of some interest whether one auction can be shown to yield a higher expected revenue and/or a lower revenue variance than another. The preceding analysis informs us that, for a given set of individual object values, each bidder will bid lower in the first-price auction than in the second-price auction where his bid equals his value. But this does not permit a comparison of the expected sales prices in the two auctions. An attempt to make such a comparison would involve comparison of the second highest object value with a bid that is *less than* the highest object value. No such comparison can be made without a stronger set of assumptions than we now have. We will return to this question in Section III below.

The expected utility hypothesis does have testable implications for the mean sales price and the variance of sales prices in the second-price auction because it implies that the bidding strategy function for that auction is the identity map (2.8). Thus the predicted sales price for that auction is the second highest of the individual object values. If each individual object value is drawn independently from a known distribution, then the implied probability distribution of the sale price is that for the (N − 1)th-order statistic for a random sample of size N from that distribution.

The experimental design which we explain in Section IV incorporates the feature that each individual object value is drawn from the uniform distribution on the interval $[\underline{v}, \bar{v}]$. The probability distribution function for the (N − 1)th-order statistic for a random sample of size N from that distribution is

$$F(p) = N[(p - \underline{v})/(\bar{v} - \underline{v})]^{N-1} - (N - 1)](p - \underline{v})/(\bar{v} - \underline{v})^N. \quad (2.9)$$

Thus the predicted mean $\bar{p}_2$ and variance V_2 of the sales price are:

$$\bar{p}_2 = \int_{\underline{v}}^{\bar{v}} p\, dF(p) = \frac{(N-1)(\bar{v}-\underline{v})}{N+1} + \underline{v}; \qquad (2.10)$$

$$V_2 = \int_{\underline{v}}^{\bar{v}} (p - \bar{p}_2)^2\, dF(p) = \frac{2(N-1)(\bar{v}-\underline{v})^2}{(N+1)^2(N+2)}. \qquad (2.11)$$

Now consider the Dutch auction and let t denote the length of time the auction has been in progress. The bid "on the clock" at time t is b(t). Since the Dutch auction is a decreasing price auction, we have

$$b(t_1) > b(t_2), \qquad \text{for all } t_1, t_2 \text{ such that } t_1 < t_2. \qquad (2.12)$$

Let $H_i(b(t))$ be the ith bidder's subjective probability at the beginning of the auction (t = 0) that he can win the auctioned object by accepting the bid b(t). If the ith bidder accepts the bid b(t), he gains the money income $v_i - b(t)$. The utility of that income is $u_i(v_i - b(t))$. Thus the expected utility at the beginning of the auction of planning to accept the bid b(t) is

$$W_i(b(t)) = H_i(b(t))u_i(v_i - b(t)). \qquad (2.13)$$

Therefore, an optimal bidding plan for the ith bidder in the Dutch auction will be to plan to accept the bid $b(t_i^o)$ that maximizes (2.13).

The immediately preceding planning model of bidder behavior in the Dutch auction ignores the fact that that auction is a "real-time" auction in which bidders can make their decisions over time. However, given the standard behavioral assumptions that we are now using, a real-time model of bidder behavior can be shown to lead to the same conclusions about bidding in the Dutch auction as does the preceding planning model. But in order to prepare for the analysis in Section VIII that involves a real-time model of Dutch auction bidder behavior that incorporates some nonstandard behavioral assumptions, we now develop a real-time model with standard behavioral assumptions.

Suppose that the auction is in progress at time t and bidder i must decide whether to accept the bid b(t) or let the auction continue. If he accepts the bid b(t), he gains the money income $v_i - b(t)$ with utility $u_i(v_i - b(t))$. If he does not accept b(t), he has a chance to obtain the auctioned object at a lower price. Suppose that the bidder does not accept b(t) but rather lets the auction continue for one more tick of the auction clock to time $t + \Delta t$, where $\Delta t > 0$. Let $H_i(b(t + \Delta t)|b(t))$ be the ith bidder's probability that he can win the auction by accepting the bid $b(t + \Delta t)$, given the observation that the auction is still in progress at time t [and thus that he could have won the auction by accepting the bid b(t)]. Then the expected utility at time t of planning to accept the bid $b(t + \Delta t)$ is $H_i(b(t + \Delta t)|b(t))u_i(v_i - b(t + \Delta t))$. Thus the change in expected utility at time t from not accepting b(t) and planning to accept $b(t + \Delta t)$ is

$$\Delta Y_i(t) = H_i(b(t + \Delta t)|b(t))u_i(v_i - b(t + \Delta t)) - u_i(v_i - b(t)). \quad (2.14)$$

With $\Delta t > 0$, we have $H_i(b(t)|b(t + \Delta t)) = 1$; therefore, Bayes' rule and (2.14) imply

$$\Delta Y_i(t) = \frac{H_i(b(t + \Delta t))}{H_i(b(t))} u_i(v_i - b(t + \Delta t)) - u_i(v_i - b(t)). \quad (2.15)$$

We will now proceed, as in our analysis of the first-price auction, to assume differentiability of the objective function. Thus, using (2.15), we find

$$\begin{aligned} Y_i'(t) &= \lim_{\Delta t \to 0^+} \left(\frac{\Delta Y_i(t)}{\Delta t}\right) \\ &= \{[u_i(v_i - b(t))H_i'(b(t))/H_i(b(t))] - u_i'(v_i - b(t))\} b'(t). \end{aligned} \quad (2.16)$$

Assume that the auction begins at $t = 0$ and ends at $t = T$. Thus if the optimal time for bidder i to stop the auction is some t_i^o such that $t_i^o \in (0, T)$ then t_i^o will satisfy the following first-order condition.

$$\begin{aligned} 0 &= Y_i'(t_i^o) \\ &= \{[u_i(v_i - b(t_i^o))H_i'(b(t_i^o))/H_i(b(t_i^o))] - u_i'(v_i - b(t_i^o))\} b'(t_i^o). \end{aligned} \quad (2.17)$$

Note that (2.17) implies the first-order condition for maximization of (2.13) on (0, T) since $b'(t_i^o) < 0$. Therefore, the two models of bidder behavior in the Dutch auction imply the same bidding behavior: accept the bid $b(t_i^o)$ that maximizes (2.13).

We now proceed, as in the preceding analysis of the first-price auction, to assume that the bid $b(t_i^o)$ satisfies the first- and second-order conditions for a maxmium of (2.13). Also assume that $[\underline{Z}_i, \overline{Z}_i]$ is the support of the probability distribution function H_i and that the expected utility function, (2.13), is pseudoconcave. Then the preceding analysis of the first-price sealed bid auction can be interpreted so as to apply to the Dutch auction. Simply replace F_i with H_i and U_i with W_i in the appropriate equations and then the preceding analysis of the first-price auction yields the following conclusions for the Dutch auction. A bidder's expected utility-maximizing bid is less than the value to him of the auctioned object. Thus the Dutch auction is not a demand-revealing allocation mechanism. Furthermore, the amount by which the bidder's object value exceeds his optimal bid depends on his risk preferences and his expectations about his rivals' bids. Thus the Dutch auction will not generally yield Pareto-efficient allocations.

A further result of interest can be simply derived as follows. Suppose that the ith bidder believed that each of his rivals would employ the same bidding strategy in the Dutch auction as he did in the first-price auction. Then F_i and H_i would be identical and the ith bidder would be led by expected utility maximization to employ the same bidding strategy in the Dutch auction that he did in the first-price auction. Thus if every bidder believed that each of his rivals would employ the same bid function in the Dutch auction that he did in the first-price auction, then every bidder would find it in his interest to do the same. In that case the Dutch and first-price auctions would have identical quantitative characteristics as well as the common qualitative characteristics discussed above. It is in this sense that the Dutch and first-price auctions are isomorphic.

Now consider the English auction. An individual bidder will obtain a positive income from participating in the auction if and only if he can win the auction with a bid that is less than his value for the auctioned object. Thus a utility-maximizing bidder will drop out of the bidding only when the bid "on the floor" equals or exceeds his value for the object. Therefore, the strategy of remaining in the bidding competition as long as the bid on the floor does not exceed the bidder's value for the object, and of dropping out as soon as it does exceed that value, is a dominant strategy. Thus the English auction is a demand-revealing allocation mechanism that will, in general, yield Pareto-efficient allocations. Furthermore, the sales price for the auctioned object will equal the second highest of the bidders' object values plus, perhaps, a minimum bid increment. The similarity of the predicted allocations of the English auction and the second-price sealed-bid auction is the reason why those auctions are said to be isomorphic.

III. IMPLICATIONS OF THE EXPECTED UTILITY HYPOTHESIS AND THE NASH EQUILIBRIUM CONDITION

The preceding analysis has explored the implications of the expected utility hypothesis for bidder behavior. We now add to that hypothesis the additional assumption that the bidders' strategy functions satisfy a Nash equilibrium condition. That condition can be explained as follows. Let S_i be the strategy function of the ith bidder where, as above, $i = 1, 2, \ldots, N$. Now suppose bidder j knows that all bidders $i \neq j$ bid in accordance with these strategy functions and that, given this information, individual j can find no way of changing his own strategy function S_j so as to increase his expected utility. If this condition holds for $j = 1, 2,$

. . ., N, then the strategy functions S_j, j = 1, 2, . . ., N, satisfy the Nash equilibrium condition.

In Section II we found that the second-price sealed-bid and English auctions have dominant strategy equilibrium bid functions. All dominant strategy functions satisfy the Nash equilibrium condition. Therefore, our assumption here of the Nash equilibrium condition is redundant for the second-price and English auctions; it has no testable implications for bidding behavior in those auctions. In contrast, the first-price sealed-bid and Dutch auctions do not have dominant strategy equilibria. Thus, our assumption here of the Nash equilibrium condition does have testable implications for bidding behavior in those auctions.

Consider the strategy functions ψ_i, i = 1, 2, . . ., N, for the first-price auction derived in Section II above. As in statement (2.4), these functions relate the expected utility-maximizing bids b_i^o to the object values v_i. Now suppose that we can define vectors θ_i, i = 1, 2, . . ., N, that represent all individual bidder characteristics that affect the utility of money income. Thus, if $u_i(y)$ is the utility of money income y to bidder i, then we can define the utility function u as follows:

$$u_i(y) = u(y, \theta_i), \qquad i = 1, 2, \ldots, N. \tag{3.1}$$

Suppose that bidder i knows his own characteristic vector θ_i but does not know the characteristic vectors of his rivals. Further assume that bidder i believes that the characteristic vectors of his rivals are drawn independently from a known probability distribution. Finally, assume that these assumptions hold for every bidder. Then one can attempt to derive a bidding strategy function ψ, such that

$$\psi_i(v) = \psi(v, \theta_i), \qquad i = 1, 2, \ldots, N, \tag{3.2}$$

where ψ maximizes the expected utility of every bidder in the first-price auction. Such a function, if it exists, will satisfy the Nash equilibrium condition and is referred to as an equilibrium strategy function and as a Nash equilibrium bid function.

Finding equilibrium strategy functions for auctions (such as the first-price sealed-bid auction) which do not have a dominant strategy equilibrium is a considerably more ambitious undertaking than is the expected utility maximization in Section II. In order to make it tractable, authors of papers on bidding theory have assumed that all bidders have the same risk preferences; that is, they have assumed that $\theta_i = \theta$, i = 1, 2, ... N. In the seminal paper on equilibrium bidding theory by William Vickrey (1961), all bidders are assumed to be risk-neutral. Vickrey further assumes that each individual's value for the auctioned object is drawn from the uniform distribution on the interval, [0, 1]. Finally each individual is assumed to know his own value for the auctioned object but to know only the distribution from which his rivals' values are drawn. Using these assumptions, and letting the number of bidders be denoted by N, Vickrey shows that the noncooperative equilibrium bid function for the first-price auction is

$$b_i = \frac{N - 1}{N} v_i, \qquad 1 = 1, 2, \ldots, N. \tag{3.3}$$

An immediate generalization of Vickrey's analysis is provided by allowing values to be drawn from a uniform distribution on any nonempty interval, $[\underline{v}, \overline{v}]$, such that $\underline{v} \geqq 0$. In that case, the equilibrium bid function is

$$b_i = \underline{v} + \frac{N - 1}{N} (v_i - \underline{v}), i = 1, 2, \ldots, N. \tag{3.4}$$

Finally, we want to note the following about the Vickrey model. Given that the highest possible value drawing is $\overline{v}$, the highest bid that satisfies (3.4) is

$$\overline{b} = \underline{v} + \frac{N - 1}{N} (\overline{v} - \underline{v}). \tag{3.5}$$

The assumption that all bidders are risk-neutral, or alternatively that they all have the same strictly concave utility function, is very restrictive. In the present paper we build on suggestions by John Ledyard to construct an equilibrium bidding model (which we will call the "Ledyard model") that permits individual bidders to differ in their attitudes towards risk. We begin by assuming that each bidder is drawn from a population of economic agents with utility of money income functions of the form

$$u_i(y) = y^{r_i}, \tag{3.6}$$

where r_i is a random variable with probability distribution Φ on [0, 1]. Note that $(1 - r_i)$ is the Arrow-Pratt constant relative risk aversion parameter for utility function (3.6).[2] Each bidder is assumed to know his own risk aversion parameter r_i, but to know only that the risk aversion parameter for each of his rivals is drawn from the probability distribution Φ. Since Φ is *not* assumed to have a density function, it can have a mass of probability of 1. Therefore, the Ledyard model includes both risk-neutral and risk-averse bidders. Included as special cases are models where all bidders are risk-neutral and all bidders are equally (constant relative) risk-averse.

The other definitions and assumptions used in the model are as follows. The number of bidders is denoted by N and the value to bidder i of the auctioned object is denoted by v_i, as in the preceding paragraphs. For each bidder i, v_i is assumed to be drawn from the uniform probability distribution on the nonempty interval $[\underline{v}, \bar{v}]$, where $\underline{v} \geqq 0$. Each bidder knows his own object value before he submits his bid but knows only the probability distribution from which his rivals' values are drawn. We also assume that the bidders do not cooperate (i.e., collude) with each other. Finally, we assume that every bidder behaves as if the assumptions contained in this paragraph and in the immediately preceding paragraph are true.

The equilibrium bidding strategy function for this model has two parts. For bids that do not exceed $\bar{b}$, as defined in statement (3.5), the equilibrium bid function is

$$b_i = \underline{v} + \frac{N - 1}{N - 1 + r_i}(v_i - \underline{v}), \; i = 1, 2, \ldots, N. \tag{3.7}$$

This will be verified as follows. Suppose that bidder j believes that the bid of each of his rivals satisfies bid function (3.7) when it does not exceed $\bar{b}$. Then we will demonstrate that bidder j's optimal bid satisfies the same function when it does not exceed $\bar{b}$.

The v inverse of bid function (3.7) is

$$v_i = g(b_i, r_i) = \frac{N - 1 + r_i}{N - 1} b_i - \frac{r_i}{N - 1} \underline{v}. \tag{3.8}$$

Now the probability that the bid of bidder i will be less than some amount b in the range of (3.7) is the probability of drawing values of v_i and r_i which when substituted in (3.7) will yield a bid less than b. Note that statements (3.5) and (3.8) imply that $g(b_i, r_i) \leqq \bar{v}$, for all $b_i \in [\underline{v}, \bar{b}]$, for all $r_i \in (0, 1]$. Therefore, using (3.8) and the density function for v_i, we find that the probability that bidder i will bid less than b is

$$\begin{aligned} F(b) &= \int_0^1 \int_{\underline{v}}^{g(b, r_i)} [\bar{v} - \underline{v}]^{-1} \, dv_i \, d\Phi(r_i) \\ &= \frac{[N - 1 + E(r)][b - \underline{v}]}{[N - 1][\bar{v} - \underline{v}]}, \end{aligned} \tag{3.9}$$

where E(r) is the expected value of r_i.

Recall that $v_1, \ldots, v_N, r_1, \ldots, r_N$ are drawn independently. Therefore, the probability that all (N − 1) rivals of bidder j will bid less than some amount b in the range of (3.7) is $[F(b)]^{N-1}$. Let γ represent the constant bid density $[N - 1 + E(r)][(N - 1)(\bar{v} - \underline{v})]^{-1}$ in (3.9). Then $[F(b)]^{N-1}$ can be written simply as $\gamma^{N-1}[b - \underline{v}]^{N-1}$.

Now recall that bidder j has the utility of money income function y^{r_j}. If he wins the auction, he receives the money income $(v_j - b_j)$. Thus his (pseudoconcave) expected utility function of the bid b_j and the object value v_j can be written as

$$U(b_j) = \gamma^{N-1}(b_j - \underline{v})^{N-1}(v_j - b_j)^{r_j}. \tag{3.10}$$

The derivative of U is

$$U'(b_j) = \gamma^{N-1}(b_j - \underline{v})^{N-2}(v_j - b_j)^{r_j - 1} [(N - 1)(v_j - \underline{v}) - (N - 1 + r_j)(b_j - \underline{v})]. \tag{3.11}$$

Assume that $v_j > \underline{v}$; then $U(b_j)$ is positive on $(\underline{v}, v_j)$. Also, $U'(b_j)$ changes sign only once on $(\underline{v}, v_j)$ and the change in sign is from positive to negative. Thus the unique bid b_j^o that maximizes (3.10) is the value that equates the square bracket term in (3.11) to zero:

$$b_j^o = \underline{v} + \frac{N - 1}{N - 1 + r_j}(v_j - \underline{v}). \tag{3.12}$$

Thus, if bidder j believes that each of his rivals will use bid function (3.7) for bids that do not exceed $\bar{b}$, then his best strategy is to use the same bid function for bids that do not exceed $\bar{b}$. Therefore, (3.7) satisfies the Nash equilibrium condition.

The equilibrium bidding strategy function (3.7) implies that the truncated probability distribution function on p, the winning bid and sales price, is the following.

$$G_T(p) = [\gamma(p - \underline{v})]^N, \forall p \in [\underline{v}, \bar{b}], \tag{3.13}$$

where

$$\gamma = [N - 1 + E(r)][(N - 1)(\bar{v} - \underline{v})]^{-1}. \tag{3.14}$$

Thus the truncated mean $\bar{p}_T$ of the sales price is

$$\bar{p}_T = \int_{\underline{v}}^{\underline{v}+1/\gamma} p \, dG_T(p) = \frac{N(N - 1)(\bar{v} - \underline{v})}{(N + 1)[N - 1 + E(r)]} + \underline{v}. \tag{3.15}$$

Even in the absence of an explicit solution for the bid function for bids which exceed $\bar{b}$, the Ledyard model of the first-price auction has several testable implications in addition to the ones which follow from the expected utility hypothesis. Given that the Dutch auction is isomorphic to the first-price auction, these same implications apply to the Dutch auction. The testable implications of the model can be divided into the implications for individual bids which follow from (3.7) and the implications for the distribution of sales prices which follow from (3.3) – (3.5). We will focus on the latter.

The Vickrey model is the special case of the Ledyard model in which the entire mass of the probability distribution Φ is concentrated at $r = 1$, which implies $E(r) = 1$. Let $G_V(\cdot)$ be the probability distribution for the sales price and $\bar{p}_V$ be the mean sales price for the Vickrey model of the first-price auction. Setting $E(r) = 1$ in statements (3.13) – (3.15) yields:

$$G_V(p) = \left[\frac{N}{(N-1)(\bar{v}-\underline{v})}(p-\underline{v})\right]^N; \tag{3.16}$$

$$\bar{p}_V = \frac{(N-1)(\bar{v}-\underline{v})}{N+1} + \underline{v}\,. \tag{3.17}$$

Statements (3.16) and (3.17) can be used to calculate the variance of the sales price in the Vickrey model as follows:

$$\begin{aligned} V_V(p) &= \int_{\underline{v}}^{\underline{v}+1/\gamma} (p-\bar{p}_V)^2\, dG_V(p) \\ &= \frac{(N-1)^2(\bar{v}-\underline{v})^2}{N(N+1)^2(N+2)}\,. \end{aligned} \tag{3.18}$$

The strict risk-averse Ledyard model is the special case of the Ledyard model which excludes the Vickrey model; in other words, the strict risk-averse Ledyard model requires that $\Phi\,(r) > 0$ for some $r < 1$ although Φ can have a mass of probability at $r = 1$. Let $G_L(\cdot)$ be the probability distribution for the sales price and $\bar{p}_L$ be the mean sales price for the strict risk-averse Ledyard model. In the following paragraphs we will derive the relation between $G_L(\cdot)$ and $G_V(\cdot)$ and the relation between $\bar{p}_L$, $\bar{p}_V$, and $\bar{p}_2$.

Inspection of statements (3.13), (3.14), and (3.16) reveals a strong first-order stochastic dominance ordering of $G_T(\cdot)$ over $G_V(\cdot)$; that is

$$G_T(p) < G_V(p)\,\forall p \in (\underline{v}, \bar{b}]. \tag{3.19}$$

Furthermore, $G_L(\cdot)$ is identical to $G_T(\cdot)$ on $[\underline{v}, \bar{b}]$. Therefore, $G_L(\cdot)$ dominates $G_V(\cdot)$ on $[\underline{v}, \bar{b}]$. Since $G_L(\cdot)$ is a probability distribution function, it must be nondecreasing on $[\bar{b}, \bar{v}]$. Furthermore, since $G_L(\bar{b}) < 1$, we must have $G_L(p) > G_L(\bar{b})$ for some $p \in (\bar{b}, \bar{v}]$. Therefore, there is a strong first-order stochastic dominance ordering of $G_L(\cdot)$ over $G_V(\cdot)$ on $[\underline{v}, \bar{v}]$. This result will be used in Sections V and VI below.

Since $G_L(\cdot)$ agrees with $G_T(\cdot)$ on $[\underline{v}, \bar{b}]$ and increases somewhere on $(\bar{b}, \bar{v}]$, we must have $\bar{p}_L > \bar{p}_T$. Inspection of (3.15) and (3.17) reveals that $\bar{p}_T > \bar{p}_V$. Finally, inspection of (2.10) and (3.17) reveals that $\bar{p}_2 = \bar{p}_V$. Therefore, $\bar{p}_L > \bar{p}_V = \bar{p}_2$. This result will be used in Sections V and VI below.

IV. EXPERIMENTAL DESIGN

The design and execution of the experiments are shaped by the following objectives:

A. Control the procedures for conducting each experiment so that all experiments—insofar as is possible—are conducted in the same way.

This is the standard design objective of minimizing extraneous "noise" in experimental outcomes. However, this consideration becomes of amplified importance when two institutions, such as the Dutch and first-price auctions, are being compared which theoretically produce identical outcomes. In this situation in order to identify any true behavioral difference in the two auctions, the variability within first-price replications and within Dutch replications may have to be relatively small. We have attempted to achieve this by using the PLATO computer system to present programmed experimental instructions and practice examples to each subject bidder, to record all data, inconspicuously, and to enforce the appropriate market rules uniformly across replications.

B. Provide an experimental design that permits paired comparisons of the treatment effects of the different auction institutions and uses different treatment switchover sequences on paired subject groups.

Again, this is the common scientific objective of attempting to reduce error. Paired comparisons increase the power of the test while switchovers increase the credibility of the claim that any measured differences are attributable to differences in the auction institutions and not to differences in particular subject groups. But these considerations increase in importance if we encounter small, and subtle, differences among the Dutch and first- and second-price auction institutions. Consequently, all experiments consist of 30 sequential auctions: 10 Dutch (first), followed by 10 first (Dutch), and finally 10 Dutch (first) auctions. Table 1 lists all the first and Dutch switchover experiments, and Table 2 lists all the Dutch and second switchover experiments. For example, in Table 1, experimental session dfd3, representing one member of the third pair of experiments, using three bidders, consists of a 30-auction sequence of 10 Dutch, 10 first, and 10 Dutch auctions. The values v_i, for each of the three bidders in dfd3, are drawn with replacement from a uniform distribution on the interval $[\underline{v}, \overline{v}]$ for each of the 30 auctions. Session dfd3 is matched with fdf3′, the latter using a different group of three subject bidders in a first-Dutch-first 30-auction sequence but using the identical value sequences drawn randomly for the three subjects in dfd3. In Table 1 any two pairs such as 3 and 3′, 10 and 10′, and 5 and 5′ are "matched" only with respect to the value sequences applying to the different sets of N subjects in each pair. An x denotes that the subjects were experienced, i.e., had participated in a previous such experiment. For example dfd8x and fdf8′x are matched pairs of experienced subjects (N = 4).

C. Vary N *systematically across experiments so that the Vickrey and Ledyard models of noncooperative equilibrium bidding can be tested for auction markets with various numbers of rival bidders.*

For each N, we want to test some implications of the (null) Vickrey risk-neutral hypothesis against the (alternative) strict risk-averse Ledyard hypothesis.[3] In Section VI we report a Kolmogorov–Smirnov test of the hypothesis that, for each N, the frequency distribution of winning bids came from the distribution function $G_V(\cdot)$ in statement (3.16). Also reported in Section VI is a binomial test, for each N, which compares observed and risk-neutral theoretical prices in first-price auctions.

D. Hold constant the expected gain per bidder as N *increases so that motivation is approximately the same for any given bidder independently of the size of the bidding group in which he/she is a participant.*

It is well known that any market (or other) decision task may have significant subjective costs of thinking, calculating, deciding and transacting (Siegel, 1961; Marschak, 1968; Smith, 1976). The greater is the explicit monetary (or other) reward relative to this subjective transactions cost, which is obtained as an outcome of the decision, the more likely will maximization of this reward be the predominating influence in determining the decision. Since subjective transactions cost is not normally observable, but may be a contaminating factor in testing a theory, it can be important to attempt to control for this contamination.

That motivation may be a problem in the larger groups follows most directly from the Vickrey model. From the Vickrey bid function (3.4), if v is the highest value drawn among N bidders, then the price is

$$p = \left(\frac{N-1}{N}\right)(v - \underline{v}) + \underline{v},$$

determined by the bid of the highest bidder. Profit to the highest bidder is thus

$$\pi = v - p = \frac{p - \underline{v}}{N-1}.$$

From the mean Vickrey price in (3.17) it follows that expected profit per bidder is

$$\frac{\bar{\pi}_v}{N} = \frac{\bar{v} - \underline{v}}{N(N+1)}. \tag{4.1}$$

Table 1.

Experimental Session	*Number of Bidders N*	*Statistic*	*Mean and Variance of Ten Prices, by Institution, in Sequence*			
			Dutch	*First*	*Dutch*	*First*
*dfd3	3	Mean	1.30	2.40	1.32	
		Variance	.098	.162	.235	
*fdf3′	3	Mean		2.66	3.32	2.02
		Variance		1.092	.297	1.166
fdf10	3	Mean		2.74	2.72	2.22
		Variance		.203	1.017	.268
dfd10′	3	Mean	2.62	2.84	2.36	
		Variance	.260	.827	.436	
dfd10x	3	Mean	2.42	2.22	2.40	
		Variance	.500	.402	.420	
fdf8	4	Mean		6.06	5.46	4.74
		Variance		.956	.996	5.256
dfd8′	4	Mean	5.70	5.46	3.78	
		Variance	1.10	.916	3.764	
dfd8x	4	Mean	5.43	6.03	5.64	
		Variance	1.329	.669	1.596	
fdf8′x	4	Mean		5.91	5.97	5.64
		Variance		1.361	.969	1.636
dfd9	5	Mean	7.75	9.52	8.83	
		Variance	3.565	.724	2.649	
fdf9′	5	Mean		8.62	9.58	9.31
		Variance		2.804	2.104	1.481
fdf9x	5	Mean		9.07	7.66	9.70
		Variance		.769	1.766	.260

Table 1. (Continued)

Experimental Session	Number of Bidders N	Statistic	Mean and Variance of Ten Prices, by Institution, in Sequence			
			Dutch	First	Dutch	First
dfd9′x	5	Mean	9.04	8.62	9.82	
		Variance	1.216	2.204	1.104	
*dfd2	6	Mean	12.72	12.48	13.60	
		Variance	3.231	6.20	3.896	
*fdf2′	6	Mean		13.60	13.44	13.96
		Variance		2.207	6.356	3.716
*dfd4	6	Mean	12.86	13.22	12.38	
		Variance	1.26	4.651	5.086	
*fdf4′	6	Mean		13.18	13.42	12.86
		Variance		2.846	5.548	6.238
*dfd5	9	Mean	31.30	30.40	29.50	
		Variance	2.56	6.26	4.00	
*fdf5′	9	Mean		31.78	30.16	30.88
		Variance		4.064	6.196	4.404

Notes:
* Initial series of experiments.
All variances are maximum likelihood estimates.

Hence, expected profit per bidder declines inversely with N^2, and motivation may decline rapidly. We attempt to control for this with the following variable reward design: For given $\underline{v}$, use (4.1) to choose $\overline{v}$ as a function of N such that expected profit per bidder is a constant v_0 under replication with different N. This requires

$$\overline{v} = N(N + 1)\, v_0 + \underline{v}\,. \tag{4.2}$$

The parameter values $v_0 = \$.40$ and $\underline{v} = \$.10$ are used in the experiments reported here. The corresponding values of $\overline{v}$, $\overline{\pi}_v$, and $\overline{p}_v$ for each N are shown in Table 3. With expected profit per bidder set at \$.40, the expected earnings of a subject in any 30-auction sequence is \$12.00. Since

Table 2.

Experimental Session	*Number of Bidders N*	*Statistic*	*Mean and Variance of Ten Prices by Institution, in Sequence*			
			Dutch	*Second*	*Dutch*	*Second*
sds7	3	Mean		2.06	2.22	1.84
		Variance		1.35	.668	.427
dsd7′	3	Mean	3.16	2.02	2.82	
		Variance	.392	.50	.304	
dsd1	6	Mean	12.78	11.66	11.94	
		Variance	2.028	12.638	3.065	
sds1′	6	Mean		11.74	13.10	10.30
		Variance		6.949	7.831	8.720
dsd3x	6	Mean	14.62	11.14	13.54	
		Variance	.571	4.487	2.674	
dsd4	9	Mean	25.90	27.10	28.72	
		Variance	10.08	12.32	7.684	
sds4′	9	Mean		27.16	29.98	26.80
		Variance		19.716	11.664	23.94

Note: All variances are maximum likelihood estimates.

subjects receive $3 for volunteering and arriving on time for an experiment, total expected earnings is $15.00 per subject per session. A session requires about 1 hour to complete.

We do not argue that making v_0 a design constant guarantees equal motivation across experiments in which N varies from 3 to 9. Rather, we argue that this procedure should yield more uniform motivation than if we ignored the issue. Ideally we want the utility of the monetary rewards relative to that of nonmonetary factors to be invariant across experiments, but neither utility nor the nonmonetary factors are observable.

In our experimental design we planned to conduct experiments for N = 3, 6, and 9 and to use these observations to test the Vickrey and Ledyard models. As explained in Section V, as the research developed it became important for additional experimental observations to be obtained for N = 4 and N = 5 (see Tables 1 and 3).

Table 3. Experimental Design Parameters: Dutch and Sealed-Bid Auctions (v_0 = \$.40, $\underline{v}$ = \$.10)

N	*3*	*4*	*5*	*6*	*9*
$\bar{v} = .4N(N+1) + .1$	4.90	8.10	12.10	16.90	36.10
$\bar{\pi}_v = .4N$	1.20	1.60	2.00	2.40	3.60
$\bar{p}_v = .4N(N-1) + .1$	2.50	4.90	8.10	12.10	28.90
$\bar{v} - \bar{p}_v = .8N$	2.40	3.20	4.00	4.80	7.20
$p_0 - \bar{v} = 2\delta$*	.40	.60	.60	.80	1.20
δ (\$/tick)	.20	.30	.30	.40	.60
τ* (seconds/tick)	2	2	2	2	2

Note:
* These entries apply to the Dutch auction only.
All other entries apply to the first, second and Dutch auctions.

E. Control for the effect of certain technical differences between Dutch and sealed-bid auctions that might account for differences in behavior other than that which would be attributable to the informational and incentive differences among the various auction institutions.

The Dutch auction requires three technical parameters to be specified which are not part of any theory of Dutch auctions and which represent typical features of an institution that are usually ignored (perhaps justifiably) in economic modeling. These parameters, which could conceivably affect behavior, are as follows:

1. The distance between the starting price and the highest possible value that might exist among the bidders. If p_0 is the starting price this distance is $p_0 - \bar{v}$ in the experiments, where distance is measured in dollars.
2. The delay time τ between price decrements, or successive "ticks" of the digital clock.
3. The decrement, δ, by which price falls with each "tick" of the clock.

If any of these parameters of the Dutch auction affect behavior, then there is not one Dutch institution but many depending upon the values of these parameters. In our initial design with experimental groups of size 3, 6, and 9, we elected to set δ at the corresponding values \$.20, \$.40. and \$.60, and clock speed ($1/\tau$) constant with τ = 2 seconds per tick (see Table 3). With these parameter values the distance $\bar{v} - \bar{p}_v$ from the greatest value to the mean Vickrey price is 12 ticks or price decrements. With similar motivation we elected to make each starting price 2 ticks or price decrements above $\bar{v}$, as shown in Table 3 for N = 3, 6, and 9. Later when we decided to conduct experiments for N = 4 and 5, some of this modular-3 symmetry could not be maintained except as an approximation. For example, δ had to be divisible by 0.10, which was the atomic measure of value chosen for the computer program.[4] Thus for both N = 4 and N = 5 we set δ = 0.30 (instead of 26.67 and 33.33, respectively, which would have preserved the modular-3 symmetry).

In the Dutch auction the price change decrement δ also defines the distance between adjacent feasible discrete bids, and therefore the prices, that can result.[5] It follows that if strict technical comparability among the experimental Dutch and sealed-bid auctions is to be maintained, it is essential that all adjacent feasible sealed-bids also be separated by the same distance δ that applies in the Dutch auction. Our computerized first- and second-price auction experiment accomplishes this by rounding each subject's bid to the nearest δ bid node, except that bids of $\underline{v}$ and of zero are always admissible. Consequently, in an auction with N = 3, since δ = .20 and values range from $\underline{v}$ = .10 to $\bar{v}$ = 4.90 (see Table 3), a bid of \$4.54 would be rounded to \$4.50, then displayed to the subject along with a message asking him/her to either confirm and enter this bid or press a key to alter it. Hence, each subject in first- or second-price sealed-bid auctions always had the opportunity to verify his/her rounded bid before it was entered into the market. Except for the starting price and clock speed parameters, all other design parameters in Table 3 apply also to the first- and second-price auctions.

Before each session begins, the experimenter executes an initialization procedure to define the experiment. In this procedure one chooses the auction sequence, say 10 Dutch, 10 first, and 10 Dutch auctions. Then the vector of parameters ($\underline{v}$, $\bar{v}$, δ, N, p_0, τ) is selected. The parameters (p_0, τ) apply only to the Dutch sequences. All other parameters apply to both the Dutch and first-price (also the second-price when appropriate) auctions. Consequently, it is impossible to do experiments in which the set of Dutch auction price outcomes is distinct from the set of first-price (or second-price) outcomes in paired comparison treatments. The PLATO instructions for a Dutch followed by a first experiment are reproduced in the Appendix to this chapter.

V. OVERVIEW OF EXPERIMENTAL RESULTS

A. Previous English and Dutch Oral Auction Experiments

Coppinger, Smith and Titus (1980, pp. 6–10) have reported the results of six English and/or Dutch oral auction experimental sessions with N =

8 bidders. In the first four of these sessions individual valuations were equally spaced, with adjacent values separated by \$1.50, but randomly assigned to individual bidders. The mean deviation of English auction prices from the second highest valuation was only \$.0124, which was insignificantly different from zero (t_e = .096). The Dutch oral auctions, in this context, yielded a mean deviation, from the second highest valuation, of −\$1.14, which was quite significantly below zero (t_d = −5.09). In sessions 5 and 6 the valuation assignments were drawn (with replacement) from the interval [\$.10, \$10.0] (using \$.10 increments). In neither auction was the mean price significantly different from the Vickrey mean given by Equation (3.17) (t_e = −1.06, t_d = −.83). Similarly, the English and Dutch price variances were not significantly different from the predictions of Equations (2.11) and (3.18). In these sessions 97.2% of the English auction awards, but only 77.8% of the Dutch auction awards, were Pareto-optimal.

In the Dutch experiments, the experimenter lowered the price in decrements of \$.50 every 7 seconds, while in the English auctions the subjects named the amount of any increase over the previous bid, which was typically \$.25. This technical difference could have affected the results, which is why this factor was computer controlled in the Dutch and first- and second-price comparisons reported here.

B. Dutch, First-Price, and Second-Price Auction Results

The means and maximum-likelihood variances for the individual Dutch-first experiments are contained in Table 1. A comparative examination of the means suggests that Dutch auction prices tend to be lower than prices in first-price sealed-bid auctions. Several of the paired experiments illustrate the importance of a paired comparison design which uses the same random sequence of valuations. Thus in fdf8 the final series of first-price auctions has a lower mean (4.74) than the middle sequence of Dutch auctions (5.46). But if we compare means using the more relevant paired experiment, dfd8′, it is seen that the final series of Dutch prices has a mean (3.78) considerably below the mean of the final first-price sequence in fdf8. In this particular matched pair, the random sequence of valuations for auctions 21 to 30 happened to be particularly low. Similarly, the mean price in first (8.62) is below both Dutch means in dfd9′x, but it is well above its matched Dutch mean (7.66) in fdf9x. Again the same phenomenon is illustrated in dfd5 and fdf5′. One cannot overemphasize the importance of a suitably controlled paired-comparison design when comparing different exchange institutions, particularly when the theory, or other a priori considerations such as exploratory experiments, allege that the institutions are equivalent.

The means and maximum-likelihood variances for the individual Dutch-

Table 4. Theoretical and Pooled Means and Variances: All Auctions

N	*Statistic*	*Dutch, Observed*	*First, Observed*	*First, Dutch, Theoretical**	*Second* *Observed*	*Second* *Theoretical*
3	Mean	2.42	2.44	2.5	1.97	2.5
	Variance	.421	.589	.384	.759	.96
4	Mean	5.33	5.64	4.9		
	Variance	1.63	1.80	.96		
5	Mean	8.78	9.14	8.1		
	Variance	2.06	1.37	1.83		
6	Mean	13.12	13.22	12.1	11.21	12.1
	Variance	3.77	4.31	3.0	8.20	6.4
9	Mean	29.26	31.02	28.9	27.02	28.9
	Variance	7.03	4.91	8.38	18.66	18.85

Note:
* These are the means and variances implied by the Vickrey hypothesis; they are calculated from (3.17) and (3.18).

second experiments are shown in Table 2. A pronounced tendency for prices in the second-price sealed-bid auction to be below those in the Dutch auction seems evident.

The pooled mean and variance of prices across all experiments for Dutch and first- and second-price auctions is shown in Table 4. For all N, the observed means are ordered $m_2 < m_d < m_1$. The theoretical prediction under the Vickrey assumptions is $m_2 = m_d = m_1$ and under the Ledyard assumptions is $m_2 < m_d = m_1$. Hence, the data appear to be consistent with the assumption that bidders are risk-averse, but inconsistent with the hypothesis that the Dutch and first-price institutions are isomorphic. These results are generally consistent with those reported by Coppinger, Smith and Titus (1980, pp. 21–22) for their less rigorously controlled experiments.

VI. TESTING THE VICKREY AND LEDYARD MODELS

Initially our research design for the Dutch and first-price auctions consisted of the eight experimental sessions indicated by an asterisk in Table 1. The remaining experiments listed in Table 1 were conducted after examining the data from the first-price auctions in this initial series. Our reasons for scheduling the additional 11 experiments will be made clear

Table 5. The Kolmogorov-Smirnov Statistic D_n^+ for Dutch, First- and Second-Auction Price Distributions

N	*First-Price Auctions,* $D_n^+ = -sup[G_n^f(p) - G(p)]$	*Dutch Auctions,* $D_n^+ = -sup[G_n^d(p) - G(p)]$	*Second-Price Auctions,* $D_n^+ = -sup[F_n^s(p) - F(p)]$
3	.09 (n = 70)	.11 (n = 120)	.33 (n = 30)*
4	.42 (n = 60)*	.37 (n = 60)*	
5	.30 (n = 60)*	.25 (n = 60)*	
6	.38 (n = 60)*	.26 (n = 110)*	.19 (n = 60)
9	.40 (n = 30)*	.19 (n = 60)	.24 (n = 30)

Note:
* Reject hypothesis that G or F is the appropriate distribution (Pr = .005).

as we examine the test results for the first-price auction reported in Tables 5 and 6.

A. Risk Aversion and Bidding Behavior in the First-Price Auction

Table 5 shows the results of a one-tailed Kolmogorov–Smirnov test, for each N, of the hypothesis that first-price auction prices came from a population with distribution given in (3.16). We reject the null hypothesis (Pr. = 0.005) in favor of the risk-averse altermative for N = 6 and 9. In applying this test to the data from the first series of N = 3 experiments we were not able to reject the null hypothesis.

The tests in Table 5 do not make use of the fact that the assigned valuations $v_i(t)$ are controlled and observed in the experiments. An alternative (and more powerful) test making use of this information involves

Table 6. Binomial Test Comparing Observed and Predicted Risk-Neutral Prices in First-Price Auctions

N	*Total Number of Auctions*	*Number of Auctions for which* $\delta(t) > 0$*	*Unit Normal Deviate,* U_p
3	70	43	1.91 ($P = .06$)
4	60	54	6.20 ($P < .0001$)
5	60	60	7.75 ($P < .0001$)
6	60	52	5.68 ($P < .00001$)
9	30	30	5.48 ($P < .0001$)

Note:

* $\delta(t) = [p(t) - \underline{v}] - \left(\frac{N-1}{N}\right) \max_i [v_i(t) - \underline{v}]$

an examination of the distribution of the difference between the observed normalized price p(t) − $\underline{v}$ and the predicted risk-neutral normalized price from (3.17), namely

$$\left(\frac{N-1}{N}\right)[\max_i v_i(t) - \underline{v}] .$$

Table 6 reports the results of a binomial test, for each N, of the null hypothesis that in first-price auctions the difference

$$\delta(t) = [p(t) - \underline{v}] - \left(\frac{N-1}{N}\right)[\max_i v_i(t) - \underline{v}]$$

is equally likely to be positive or negative (risk-neutral) against the one-tailed alternative that the difference is more likely to be positive (risk-averse). From the values of U_p (the unit normal approximation to the binomial) we are able to reject the null hypothesis at very high levels of significance for N = 6 and 9. In applying this test to the data from the first series of N = 3 experiments we were not able to reject the null hypothesis at any level of significance approaching that of the N = 6 and 9 tests.

The apparent divergence of the N = 3 results from the N = 6 and 9 results in both the Kolmogorov–Smirnov and binomial tests led to three provisional hypotheses:

(1) The Ledyard model is superior to the Vickrey model, except that for N = 3 the assumption of noncooperative (Nash) behavior fails.
(2) The Ledyard model is superior to that of Vickrey for values of N larger than 4 or 5, i.e., the assumption of noncooperative behavior breaks down somewhere between N = 3 and N = 6, to be determined.
(3) The Ledyard model is superior to the Vickrey model for all values of N, with the apparent failure for N = 3 attributable to sampling error in our first two experiments.

On the basis that there is considerable experimental evidence in the context of oligopoly competition (Shubik 1975, p. 282; Fouraker and Siegel, 1963) to suggest that the assumption of noncooperative behavior is supported for N ≥ 3, we conjectured that the additional experimental observations would support (1). However, on this same prior evidence we could not rule out the possibility that (3) would be supported. At this critical juncture the additional experiments were conducted. These experiments included sixteen sequences of ten first-price (and seventeen Dutch) auctions and are recorded in Table 1 without asterisks.

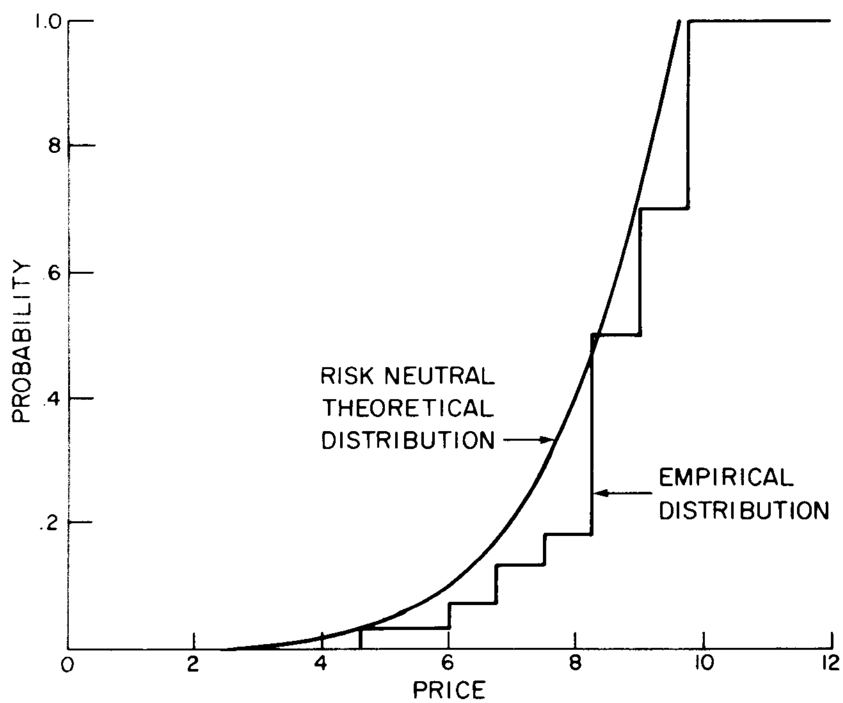
The test results in Tables 5 and 6 allow us to reject the null hypotheses for N = 4 and 5 at the same levels of significance at which we were able to reject the null hypotheses for N = 6 and 9. The theoretical and empirical distributions for the Kolmogorov–Smirnov test for N = 5 are presented in Chart 1. On the basis of these tests, for N > 3, we reject the risk-neutral in favor of the risk-averse model of Nash equilibrium behavior.

The same tests applied to data from all of the N = 3 experiments reveal, in Tables 5 and 6, that we cannot reject the risk-neutral hypothesis. However, since there is no reason to suppose that individuals in groups of size N = 3 are any less risk-averse than those in groups of size N > 3, we interpret the results as also supporting the subsidiary hypothesis that the assumption of noncooperative behavior fails to apply when N = 3. We can think of no alternative explanation.

B. Risk Aversion and Bidding Behavior in the Dutch Auction

Table 5 reports the results of a one-tailed Kolmogorov–Smirnov test for each N, of the hypothesis that Dutch auction prices came from a

Chart 1. First-Price: Theoretical and Empirical Probability Distributions for N = 5



population with distribution given in (3.16). We are able to reject the null hypothesis (Pr. = 0.005) in favor of the risk-averse alternative for N = 4, 5, and 6 but are not able to do so for N = 3 and 9. The N = 3 Dutch auction results are consistent with the N = 3 first auction results. But the Dutch auction test results for N = 9 are not consistent with the pattern of test results for the first-price auction. However, one must keep in mind that $G_V(\cdot)$, for the Vickrey hypothesis, or $G_L(\cdot)$, for the Ledyard hypothesis, is the distribution function for sales price in the Dutch auction only if it is isomorphic to the first-price auction. Thus the Dutch auction test results in Table 5 are for a joint test for the effects of risk aversion and the first/Dutch isomorphism. Our rejection of this isomorphism in the next section of the paper will clarify the Dutch auction test results for N = 9 in Table 5.

C. Bidding Behavior in the Second-Price Auction

Now consider the second-price auction and note that the pooled mean price in second-price auctions reported in Table 4 is below the theoretical mean in Equation (2.10). Results of the Kolmogorov–Smirnov test of the hypothesis that second-price winning bids came from a population with distribution given by (2.9) are shown in Table 5. The hypothesis is rejected for N = 3, but not for N = 6 and 9. Coppinger, Smith and Titus (1980) also report mean prices below the dominant strategy expected price in second-price auctions (with varying significance in different experiments). They also report considerable learning effects in that some subjects in second-price auctions converge to the dominant strategy over successive auctions.

VII. PRICE AND EFFICIENCY COMPARISONS IN THE DUTCH, FIRST-PRICE AND SECOND-PRICE AUCTIONS

The paired sample comparisons listed in Table 7 show that the Dutch and first-price auctions are not behaviorally isomorphic. In every paired comparison the mean price is higher in the first than in the Dutch auction. Using the nonparametric sign test, we reject the null hypothesis that the mean price difference $m_1 - m_d$ is as likely to be positive as negative in favor of the alternative that Dutch prices tend to be below those in the first-price auction (Pr < .001).

These Dutch auction findings not only call into question the theoretical equivalence of Dutch and first-price auctions; they also provide clear evidence against the proposition that Dutch prices will be among the highest obtainable on the grounds that any buyer will tend to "stop the clock" as soon as the price is slightly below that buyer's reservation

Table 7 Dutch-First Paired-Sample Mean Price Differences

Experiment	*N*	*First Price*	*Dutch Price*	$m_f - m_d$
dfd 3, fdf 3′	3	2.36	1.98	.38
fdf 10, dfd 10′	3	2.60	2.57	.03
fdf 8, dfd 8′	4	5.42	4.98	.44
dfd 8x, fdf 8′x	4	5.86	5.68	.18
dfd9, fdf 9′	5	9.15	8.72	.43
fdf 9x, dfd 9′x	5	9.13	8.84	.29
dfd 2, fdf 2′	6	13.35	13.25	.10
dfd 4, fdf 4′	6	13.09	12.89	.20
dfd 5, fdf 5′	9	31.02	30.32	.70

value (Boulding, 1948, p. 42; Cassady, 1967, p. 67). The opposite behavior is evident. On a more impressionistic level it is worth noting in this regard that many subjects report that they enjoy the "clock experiment" more than the others because of the "suspense of waiting." In this sense they seem to perceive the Dutch auction as a "waiting game," in which lower bids are entered than in the first-price sealed-bid auction.

The Dutch and second-price paired sample comparisons are shown in Table 8. In each paired comparison the mean price is higher in the Dutch than in the second-price auction. This is consistent with the risk-averse model of Dutch auction bidding and the dominant strategy model of second-price auction bidding. It is also consistent with the more limited, indirect, empirical results reported by Coppinger, Smith and Titus (1980, pp. 9–10, 13–18) in which Dutch prices are above English auction prices and the latter do not differ significantly from prices in the second-price auction. Using the sign test, the positive difference between the Dutch auction mean and the second-price mean is significant (Pr = .06).

If we let V_N (t) be the highest value drawn among N bidders in auction t, and $W_N(t)$ be the value drawn by the winning bidder in auction t, then the efficiency of auction t is measured by

$$E_N(t) = 100\ W_N(t)/V_N(t).$$

Table 8. Dutch-Second Paired-Sample Mean Price Differences

Experiment	*N*	*Mean Dutch Price*, m_d	*Mean Second Price*, m_2	$m_d - m_2$
sds7, dsd7′	3	2.73	1.97	.76
dsd1, sds1′	6	12.61	11.23	1.37
dsd4, sds4′	9	28.20	27.02	1.18

Table 9. Mean Efficiency (and Percent Pareto-Optimal Allocations) by Institution and Number of Bidders

Institution \ *Number of Bidders*	*3*	*4*	*5*	*6*	*9*	*Over All Groups*
Dutch	97.32 (81.82)	96.25 (76.67)	98.48 (81.67)	98.89 (83.64)	98.44 (71.67)	97.95 (80.00)
First Price	97.61 (82.86)	99.62 (95.00)	99.80 (93.33)	98.26 (83.33)	99.77 (83.33)	98.88 (87.86)
Second Price	99.28 (93.33)			99.89 (96.67)	99.54 (90.00)	99.65 (94.00)

Efficiency is 100% if and only if the winning bidder drew the highest value, which is a Pareto-optimal allocation. Unrealized gains from exchange will characterize any auction which is less than 100% efficient. Table 9 reports the mean efficiency of all auctions classified by institution and group size. The most efficient institution is the second-price auction with mean efficiency 99.65% over all groups, followed by the first-price auction (98.88%), with the Dutch auction being the least efficient (97.95%). Table 9 also reports (in parentheses) the percent of total auctions that were Pareto-optimal allocations.

VIII. TWO BIDDING MODELS THAT ARE CONSISTENT WITH BIDDING BEHAVIOR IN THE DUTCH AND FIRST-PRICE AUCTIONS

In Section II we showed that standard behavioral assumptions imply that the Dutch and first-price auctions are theoretically isomorphic. Therefore, if one is to construct a bidding theory that is consistent with the observation that these auctions are *not* behaviorally isomorphic, he must incorporate some nonstandard behavioral assumptions in the model. We will in this section provide two possible explanations of the failure of the predicted isomorphism. One explanation will be based on the utility of playing the Dutch auction "waiting game." The other explanation will be based on bidder violation of Bayes' rule.

We have adopted the "utility of playing the game" approach to modeling the Dutch auction because of the comments made by some experimental subjects. They reported that they especially enjoyed the "clock experiment" more than the others because of the "suspense of waiting." We inferred from these comments that the Dutch and first-price auctions may not be behaviorally isomorphic because of a property that follows from the real-time aspect of the Dutch auction: bidder utility from playing the waiting game.

We have two reasons for adopting the approach to the Dutch auction that is based on bidder violation of Bayes' rule. First, there is independent evidence that behavior is not consistent with Bayes' rule (Grether, 1980). Secondly, the only predicted comparison of bidding behavior across auctions that depends on Bayes' rule is the predicted isomorphism between the Dutch and first-price auctions, and that prediction is the one that is clearly inconsistent with our observations.

Now consider bidding models that include utility from playing the game. Assume that in the first-price auction an active bidder i gets the nonnegative utility a_i from playing the first-price auction game. That is, we now replace the expected utility function (2.1) with

$$U_i(b_i) = a_i + F_i(b_i)u_i(v_i - b_i), \tag{8.1}$$

where $a_i \geq 0$. We now proceed, as we did with (2.1), to assume that (8.1) is pseudoconcave and has a unique interior maximum at b_i^o. Then b_i^o will satisfy the first-order condition,

$$\begin{aligned} 0 &= U_i'(b_i^o) \\ &= F_i'(b_i^o)u_i(v_i - b_i^o) - F_i(b_i^o)u_i'(v_i - b_i^o). \end{aligned} \tag{8.2}$$

Now consider the Dutch auction and assume that the bidder gets utility $\alpha_i(t)$ from playing a Dutch auction game of length t. Assume that α_i is a positive, increasing function; that is, assume that the bidder enjoys playing this "waiting game" and that he gets more utility from playing a longer game. Suppose that the auction is in progress at time t and bidder i must decide whether to accept the bid b(t) or let the auction continue. If he accepts the bid b(t) he gains the money income $v_i - b(t)$ with utility $u_i(v_i - b(t))$. In addition, he gains the utility $\alpha_i(t)$ from playing a Dutch auction game of length t. If bidder i does not accept b(t), he will be able to play a longer auction game and will have a chance to obtain the auctioned object at a lower price. Suppose that the bidder does not accept b(t) but rather lets the auction continue for one more tick of the auction clock to time $t + \Delta t$, where $\Delta t > 0$. By doing so, he obtains the utility $\alpha_i(t + \Delta t)$ of playing a longer game and the probability $H_i(b(t + \Delta t)|b(t))$ of obtaining the utility $u_i(v_i - b(t + \Delta t))$ of the monetary gain $v_i - b(t + \Delta t)$. Thus the change in expected utility at time t from not accepting b(t) and planning to accept $b(t + \Delta t)$ is

$$\begin{aligned} \Delta X_i(t) &= \alpha_i(t + \Delta t) - \alpha_i(t) \\ &\quad + H_i(b(t + \Delta t)|b(t))u_i(v_i - b(t + \Delta t)) - u_i(v_i - b(t)). \end{aligned} \tag{8.3}$$

Assume (for now) that bidder i believes that his rivals will use the same bidding strategies in the Dutch and first-price auctions. Given Bayes' rule, this assumption implies

$$H_i(b(t + \Delta t)|b(t)) = F_i(b(t + \Delta t))/F_i(b(t)). \tag{8.4}$$

We now proceed, as in Section II, to assume differentiability of the objective function. Thus, using (8.3) and (8.4), we find

$$\begin{aligned} X_i'(t) &= \lim_{\Delta t \to 0^+} \frac{\Delta X_i(t)}{\Delta t} \\ &= \alpha_i'(t) + \{[u_i(v_i - b(t))\, F_i'(b(t))/F_i(b(t))] \\ &\qquad - u_i'(v_i - b(t))\}\, b'(t). \end{aligned} \tag{8.5}$$

Suppose that the optimal time for bidder i to stop the Dutch auction is some t_i^{**} such that $t_i^{**} \in (0, T)$. Then, using (8.5), we have

$$\begin{aligned} 0 &= X_i'(t_i^{**}) \\ &= \alpha_i'(t_i^{**}) + \{[u_i(v_i - b(t_i^{**}))F_i'(b(t_i^{**}))/F_i(b(t_i^{**}))] \\ &\qquad - u_i'(v_i - b(t_i^{**}))\}\, b'(t_i^{**}). \end{aligned} \tag{8.6}$$

We have $\alpha_i'(t_i^{**}) > 0$ and $b'(t_i^{**}) < 0$; therefore the curly bracket term in (8.6) must be positive. But this implies that the derivative of (8.1) is positive at $b(t_i^{**})$. Therefore, since (8.1) is pseudoconcave, we have $b(t_i^{**}) < b_i^o$, that is, for a given object value v_i, bidder i's optimal stopping time for the Dutch auction yields a bid that is less than his bid in the first-price auction.

If bidder i will bid less in the Dutch auction than in the first-price auction, then he might believe that his rivals will behave in the same way. Suppose that is the case; specifically, assume that

$$H_i(b(t + \Delta t)|b(t)) = [F_i(b(t + \Delta t))/F_i(b(t))]^{\theta_i}, \tag{8.7}$$

where $\theta_i < 1$. Statement (8.7) implies a first order stochastic dominance ordering of the two distributions. Let $\hat{t}_i$ be the ith bidder's optimal stopping time for the Dutch auction when his expectations satisfy (8.7) rather than (8.4); then we have

$$\begin{aligned} 0 &= \alpha_i'(\hat{t}_i) + \{[\theta_i u_i(v_i - b(\hat{t}_i))\, F_i'(b(\hat{t}_i))/F_i(b(\hat{t}_i))] \\ &\qquad - u_i'(v_i - b(\hat{t}_i))\}\, b'(\hat{t}_i). \end{aligned} \tag{8.8}$$

We have $\alpha_i'(\hat{t}_i) > 0$ and $b'(\hat{t}_i) < 0$; therefore the curly bracket term in (8.8) must be positive. But this implies that the derivative of (8.1) is positive at $b(\hat{t}_i)$ and therefore that $b(\hat{t}_i) < b_i^o$ since (8.1) is pseudoconcave. Thus, bidder i will bid less in the Dutch auction than in the first-price auction.

Now consider the model of bidder behavior in the Dutch auction that includes bidder violation of Bayes' rule. Consider again the "real-time" model of behavior in the Dutch auction that was developed in Section II. Observation of rivals' bidding behavior in the Dutch auction is *not* informative (in the specific sense in which that term is used in statistics). In fact, recognizing that the Dutch auction is not informative is one avenue for understanding the isomorphism between the Dutch and first-price auctions that holds under conventional assumptions.

Now, however, assume that our representative bidder i behaves as if his observations of his rivals' bidding behavior in the Dutch auction were informative. Specifically, assume that bidder i violates Bayes' rule in the following way. Given the observation that none of his rivals has accepted a bid that is greater than or equal to b(t), the expected utility-maximizing bidder will utilize the probability $G_i(b(t + \Delta t)|b(t))$ that none of his rivals will accept a bid that is greater than or equal to $b(t+\Delta t)$, where $\Delta t > 0$. Assume that bidder i violates Bayes' rule in a way such that

$$\begin{aligned} G_i(b(t + \Delta t)|b(t)) &= [H_i(b(t + \Delta t)|b(t))]^{\pi_i} \\ &= [H_i(b(t + \Delta t))/H_i(b(t))]^{\pi_i}, \qquad \pi_i < 1. \end{aligned} \tag{8.9}$$

Statement (8.9) implies a first order stochastic dominance ordering of the ith bidder's conditional probability distribution $G_i(b(t + \Delta t)|b(t))$ and the conditional probability distribution implied by Bayes' rule, $H_i(b(t + \Delta t))/H_i(b(t))$. Thus (8.9) implies that, having observed that none of his rivals has accepted a bid that is greater than or equal to b(t), the bidder underestimates the risk he bears by continuing to let the auction clock run.

We could now proceed to show that if we incorporate (8.9) into the "real-time" model of bidder behavior in the Dutch auction, then the optimal bid in the Dutch auction is less than the optimal bid in the first-price auction. This conclusion follows in the case where the representative bidder believes that his rivals will use the same bidding strategies in the Dutch and first-price auctions and in the case where he believes that his rivals will bid less in the Dutch auction, as in (8.7). The reasoning that leads to these conclusions will not be reproduced here because it is essentially the same as the preceding argument that includes statements (8.7) and (8.8).

Given two theories, each predicting Dutch prices to be lower than prices in the first-price sealed-bid auction, one would like to be able to design an experiment that would provide a test of the two models of behavior. The simplest such experiment that we can suggest is to replicate an existing set of dfd and fdf experiments, with all parameters unchanged with the exception that the monetary reward level is doubled. If the utility-of-"suspense" model is a correct interpretation of the Dutch auction results, then doubling the reward level should cause Dutch prices to increase toward the level of prices in the first-price auction. If the probability-miscalculation model is appropriate, the price discrepancy in the two auction systems should remain unchanged.

IX. SUMMARY AND CONCLUSIONS

A. Our theoretical analysis of Dutch and English "oral" auctions and of first- and second-price sealed-bid auctions, generalizing and extending the work of Vickrey (1961), can be summarized as follows:

1. Implications of the expected utility hypothesis: In first-price auctions the optimal individual bid is less than the value of the auctioned item. The amount by which value exceeds the optimal bid depends upon an individual's risk preference u_i and expectations F_i about rival bidding behavior. Because of differing risk preferences and expectations among individuals, bids need not be ranked in the same order as individual values, and thus allocations need not be Pareto-efficient. A sufficient condition for allocations to be Pareto-optimal is that all individuals have identical strictly increasing bid functions. This requires the untenable assumption that all bidders have the same utility function.

Bayes' rule implies that the conclusions from the analysis of the first-price auction apply to the Dutch auction. This is because individual Bayesian expectations about rival bidding behavior will not be affected by the "knowledge" that at any given time in the auction no bidder has as yet "stopped the clock." Such expectations are completely determined by the prior probability that a given bid will win, and the Dutch clock process itself is noninformative.

In the second-price auction the optimal bid is equal to the value of the auctioned object independent of risk preference and expectations about rival bids (a bid equal to value is a dominant strategy). The optimal bid is thus higher in the second-price auction than in the first-price auction for any given valuation. The allocations are Pareto-efficient, and the probability distribution of price is that for the $(N - 1)$th-order statistic for a random sample of size N from the probability distribution of values. A similar calculation is not possible for the first-price auction without imposing additional behavioral assumptions.

In the English auction, an expected utility-maximizing bidder will drop out of the bidding only if the outstanding bid is not less than his/her value. This is a dominant strategy, and in this sense the English and second-price auctions are isomorphic.

2. Implications of the expected utility and Nash equilibrium hypotheses: By adding the assumption of Nash equilibrium bidding behavior and specializing the expected utility hypothesis to the class of utility functions with constant relative risk-aversion, we can deduce the optimal bid as a linear function of object value in the first-price auction for bids which do not exceed the risk-neutral maximum bid, $\bar{b}$. The resulting model does not imply Pareto-optimal allocations but does make possible derivation of the truncated probability distribution of selling price. The Vickrey risk-neutral model of the first-price auction is the limiting case in which all bidders have a zero coefficient of relative risk aversion.

B. From the results of 780 Dutch, first- and second-price auction experiments we offer the following conclusions concerning the market price behavior, and Pareto-efficiency of these three institutions:

1. Pooling across all experiments, the mean price in second-price auctions is less than the mean Dutch price, which is in turn less than the mean in first-price auctions. This is consistent with the weak (qualitative) implications of the expected utility hypothesis, but not the hypothesis of Bayes' rule used in the analysis of Dutch auctions.
2. In first-price auctions for groups of size N = 4, 5, 6, and 9, but not for N = 3, we reject the null hypothesis of risk-neutral Nash equilibrium bidding behavior in favor of our version of the Ledyard risk-averse model of Nash bidding behavior. This conclusion is supported by Kolmogorov–Smirnov tests on the frequency distribution of prices and by binomial tests comparing observed and risk-neutral theoretical prices.
3. The Dutch and first-price sealed-bid auctions are not isomorphic. This conclusion receives its strongest support from experiments carefully designed for paired comparison, in which mean Dutch prices are consistently and significantly below mean first-prices. We have offered two theoretical explanations for the lower observed prices in the Dutch auction. One theory postulates a utility for the "suspense of waiting" in the real-time Dutch auction. The second theory postulates a systematic underestimate of the Bayes' rule risk of loss from allowing the Dutch clock to continue. The second theory is consistent with the results of independent experiments testing Bayes' rule, while the first theory is consistent with the reported impression of subjects that they like the "suspense of waiting" associated with Dutch auctions.
4. For N = 3, but not for N = 6 and 9, we reject the hypothesis that prices in second-price auctions came from a population with distribution defined by the (N − 1)th-order statistic of values, which is implied by the dominant strategy model of bidding behavior.
5. It is conjectured that the deviant results for the case N = 3 in both first- and second-price auctions are due to failure of the assumption of noncooperative behavior which underlies both the Nash and dominant-strategy models of bidding.

6. Efficiency, measured by the percentage of the theoretical total gains from exchange that are actually realized, is greatest in second-price auctions, next highest in first-price auctions, and lowest in Dutch auctions. These results are inconsistent with the hypothesis, underlying most bidding models, that individuals have identical utility functions.

This report has concentrated on the behavior of market price and allocations and the consistency of such data with the predictions of bidding theory in the three institutions studied. Questions of individual bidding behavior, including learning with experience, will be examined in a separate paper.

APPENDIX

The instructions to follow are for a PLATO Dutch auction experiment followed by a first price sealed bid auction experiment.

Program written by Bruce E. Roberson.
Consulting provided by Vernon L. Smith.

INSTRUCTIONS

This is an experiment in the economics of market decision making. The National Science Foundation has provided funds for the conduct of this research. The instructions are simple, and if you follow them carefully and make good decisions you may earn a CONSIDERABLE AMOUNT OF MONEY which will be PAID TO YOU IN CASH at the end of the experiment.

In this experiment we are going to create a market in which you will be buyers of a fictitious commodity in a sequence of auctions. The PLATO computer will act as the "auctioneer," but it is completely passive in the sense that it is used solely to store and transmit information on decisions made by the participants in the market.

Please type in your LAST NAME after the arrow then press -NEXT-.
(Use the EDIT key if you make a typing error.)
This information is used solely to aid in the distribution of the cash earnings at the end of the experiment.

→ Testor

Thank-you, participant Testor.

Auction Number	Resale Value	Market Price	Profit
1	8.50		
2	6.00		
3	8.10		
4	12.00		
5	15.90		
6	4.70		
7	3.60		
8	13.00		
9	12.70		
10	9.50		
		Total Profit:	

This is your personal record sheet for the market experiment. Notice that the column labeled "RESALE VALUE" has been filled in with dollar and cents amounts. This indicates the value to you of purchasing a unit of this commodity. This value to you may be thought of as the amount you would receive if you were to resell the unit.

Notice that you have a resale value of $8.50 for the first auction, a resale value of $6.00 for the second auction, a resale value of $8.10 for the third auction, and so on. These resale values are assigned randomly. You have an equally likely chance of receiving any resale value between $0.10 and $16.90, inclusive. That is, you are equally likely to receive $0.10, $0.50 .... $16.50, $16.90.
Furthermore, the chance of you being assigned any particular value in this range, for example $8.50 is not changed if that value was assigned earlier to you or to another participant. It is therefore possible for you to get the same resale value for different auction periods or for two participants to have the same value in the same auction. All participants will have their resale values assigned in this manner.

If you are able to make a purchase (we'll describe the buying process soon) you will receive the difference between your resale value and the price you pay.

TO SUM UP:

resale value − price paid = profit

Note that your cash profits depend upon your ability to buy a unit at a price below the resale value given on your personal record sheet. Also note that if you buy a unit at a price equal to its resale value your profit will be zero.

Your earnings will be automatically entered in your record sheet at the close of each auction. Earnings (profits) are accumulated over several auctions, with your total profit at the end of the experiment being the summation of your profits over all auctions. But, you may ask, "How do I purchase this commodity?" Good question. Press "NEXT" for the answer.

Auction Number	Resale Value	Market Price	Profit
1	8.50		
2	6.00		
3	8.10		
4	12.00		
5	15.90		
6	4.70		
7	3.60		
8	13.00		
9	12.70		
10	9.50		
	Total Profit:		

8.90

confirm

This is the starting price of the auction, in this example. It will be $17.70 in the experiment.

This is how your screen will look during the experiment. The "clock" will act as an auctioneer.

Notice that there is a dollar and cents amount inside the clock. This is the price the auction will start at. Every 2 seconds the price in the clock will decrease by $0.40.
When all of the participants are ready to begin the price in the "clock" will start to decrease. Press "NEXT" to see what this will look like.

Auction Number	Resale Value	Market Price	Profit
1	8.50		
2	6.00		
3	8.10		
4	12.00		
5	15.90		
6	4.70		
7	3.60		
8	13.00		
9	12.70		
10	9.50		
	Total Profit:		

8.50

confirm

During the experiment the price in the ""clock'' will change automatically, you will not have to press any keys. Press "NEXT" to see what this will look like.

Auction Number	Resale Value	Market Price	Profit
1	8.50		
2	6.00		
3	8.10		
4	12.00		
5	15.90		
6	4.70		
7	3.60		
8	13.00		
9	12.70		
10	9.50		
	Total Profit:		

7.30

confirm

If you wish to buy the commodity at the price shown inside the clock press the key marked "LAB" on your keyboard. Suppose you wish to accept the price which is showing right now. You would press "LAB" to accomplish this. Press the key labeled "LAB" now to see what happens.

Auction Number	Resale Value	Market Price	Profit
1	8.50		
2	6.00		
3	8.10		
4	12.00		
5	15.90		
6	4.70		
7	3.60		
8	13.00		
9	12.70		
10	9.50		
		Total Profit:	

7.30

Please confirm bid.

confirm

Notice that upon accepting you must then confirm the contract to ensure that you have not touched the "LAB" key by mistake. To confirm the contract tap the box under the clock labeled "CONFIRM." The touch panel acts like pressing a key. IN THE ACTUAL EXPERIMENT YOU MUST DO THIS WITHIN 3 SECONDS OR THE CONTRACT WILL NOT BE CONFIRMED. If you fail to confirm your clock will continue to run as before, and you may "LAB" again if no one else has purchased the unit being auctioned. Tap the confirm box now!

Auction Number	Resale Value	Market Price	Profit
1	8.50	7.30	1.20
2	6.00		
3	8.10		
4	12.00		
5	15.90		
6	4.70		
7	3.60		
8	13.00		
9	12.70		
10	9.50		
		Total Profit:	1.20

7.30

confirm

THE FIRST PERSON TO BOTH "LAB" AND CONFIRM WILL BE THE ONLY ONE TO RECEIVE THE UNIT BEING AUCTIONED! Notice that upon confirming the contract your personal record sheet was filled in for you. This will be done for you at the close of each auction period. If someone else confirms before you, stars will be entered in the column labeled "PROFIT." The winning price will be entered under "MARKET PRICE" so that you will have a record of all winning prices. Press "NEXT" to continue.

Let's review the important items. (1) You have an equally likely chance of receiving any resale value between $0.10 and $16.90 inclusive. (2) Your bid MUST be less than or equal to your resale value. (3) You may accept the "auctioneers" offer by pressing "LAB." (4) You must confirm your acceptance within 3 seconds to make a contract. (5) The first person to "LAB" and confirm will receive the unit. (6) The starting price in each auction will be $17.70.

This is the end of the instructions. If you wish to go back and examine all the instructions over again press "HELP". For a quick review press "BACK". If you wish to see the auction example and the instructions which follow press "LAB". If you feel you now understand the instructions and are prepared to proceed with the actual experiment press "NEXT". If you have a question that you feel was not adequately answered by the instructions please raise your hand and ask the monitor before proceeding. YOUR EARNINGS MAY SUFFER IF YOU PROCEED INTO THE MARKETPLACE WITHOUT UNDERSTANDING THE INSTRUCTIONS!!

Are you sure you understand the instructions? You will not be able to return to them if you proceed beyond this point. Press "NEXT" to continue or "BACK" to return to the instructions.

After 10 Dutch auctions are completed, the following instructions are administered.

In the auction periods to follow we will have different bidding rules. The highest bidder will still be the winning buyer of the unit of the commodity for sale, but the method of entering bids will change. We will use this new bidding procedure for several more auctions. Profits accumulated using this new procedure will be added to those you have already accumulated.

On your screen your table will again be displayed, and your (and everyone else's) resale values will still be selected by the same random process. The method of figuring your profit will also remain unchanged. If you are the winning bidder you will still receive the difference between your resale value and the price you bid.

Auction Number	Resale Value	Market Price	Profit
1	8.50		
2	16.50		
3	0.10		
4	4.40		
5	8.70		
6	12.90		
7	16.60		
8	0.30		
9	3.80		
10	4.20		
		Total Profit:	

Please enter your bid for auction #1
(rounded automatically to the nearest .40)

(Press "NEXT" to enter it, or edit to change it)
This is how your screen will look during the next few auction periods. Instead of the clock appearing on your screen, you will now see the above message. Your job is to attempt to purchase the unit of commodity in each auction by entering a bid for it. The highest bidder in each auction period will be awarded the unit.

Let's go through a sample auction. Given your first period resale value of $8.50 you will enter a bid for this unit. Your bid will be automatically rounded to the nearest multiple of $.40 below the value of $17.70. That is, your bid will be rounded to the nearest value such as 17.30, 16.90, 16.50, etc. Suppose you wanted to bid 7.30 for this unit. To do so type in 7.30 and then press "NEXT". Try this now. (Use the "EDIT" key if you make a typing error.)

Please enter your bid for auction #1
(rounded automatically to the nearest .40)
(Press "NEXT" to enter it, or edit to change it)
> 7.30

Press "NEXT" to confirm your bid of 7.30 or press -BACK- to change it.

Notice that upon selecting a bid you may either confirm it or enter a new bid. Suppose you are satisfied with your bid and wish to confirm it. Press "NEXT" to do this.

Auction Number	Resale Value	Market Price	Profit
1	8.50	7.30	1.20
2	16.50		
3	0.10		
4	4.40		
5	8.70		
6	12.90		
7	16.60		
8	0.30		
9	3.80		
10	4.20		
		Total Profit:	1.20

If you are the highest bidder your personal record sheet will be filled in as above. As before, if you do not receive the unit, the column labeled "PROFIT" will have stars entered into it.

The winning bid will be displayed in the column labeled "MARKET PRICE" so that you will have a record of all contract prices. In the event that two or more bidders tie for the highest bid, PLATO will select the winner at random.

Let's review the important items. (1) Your bid MUST be less than or equal to your resale value. (2) You can change your bid if you have not yet confirmed it already. (3) The highest bidder will be the only person to receive the unit.

This is the end of the instructions. If you wish to go back and examine all the instructions over again press "HELP". For a quick review press "BACK". If you wish to see the auction example and the instructions which follow press "LAB". If you feel you now understand the instructions and are prepared to proceed with the actual experiment press "NEXT". If you have a question that you feel was not adequately answered by the instructions please raise your hand and ask the monitor before proceeding. YOUR EARNINGS MAY SUFFER IF YOU PROCEED INTO THE MARKETPLACE WITHOUT UNDERSTANDING THE INSTRUCTIONS!!

Are you sure you understand the instructions? You will not be able to return to them if you proceed beyond this point. Press "NEXT" to continue or "BACK" to return to the instructions.

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NOTES

1. In correspondence (to Smith, February 21, 1980), Michael Darby has noted that, based on his personal experience, the application of second-price procedures to "book bids" is standard practice in American auctions.

2 In other words, for utility function (3.6) one has $[-yu_i''(y)/u_i'(y)] = (1 - r_i)$. On the basis of this familiar equation, we have decided to label (3.6) as a constant relative risk-averse utility function even though the interpretation does not follow when the utility function is defined on income rather than terminal wealth.

3 The risk-neutral case will be referred to as the null or Vickrey hypothesis, whereas strict risk aversion in which not all bidders are risk-neutral and $E(r) < 1$ will be called the Ledyard hypothesis. Vickrey (1961) was well aware of what would be the effect of risk aversion, but did not formally treat this case.

4 The limiting factor here was the response limits of the display screen in the Dutch auction. The PLATO (or any other) system cannot display digital changes at anything approaching a speed which is "fast" by electronic standards. For example, a grid 10 times as fine as our $.10 unit coupled with a 10-fold increase in clock speed would vastly exceed the screen's display capability (as well as the discerning power of the eye and brain).

5 That is, if at time t seconds after the beginning of the auction the clock price reads p_t, then at time $t + 2$ seconds the clock will tick down to a price $p_{t+2} = p_t - \delta$. If within the next 2 seconds a subject depresses an "accept" key and then is the first person to touch the sensitive "confirm" area on the computer screen, the clock stops at the price $p_t - \delta$, which is the winning bid.

REFERENCES

Boulding, Kenneth, *Economic Analysis*. New York: Harper, Revised Edition, 1948.

Cassady, Ralph, *Auctions and Auctioneering*. Berkeley and Los Angeles: University of California Press, 1967.

Coppinger, Vicki, Vernon L. Smith, and John Titus, "Incentives and Behavior in English, Dutch, and Sealed-Bid Auctions." *Economic Inquiry* 18(January):1–22, 1980.

Fouraker, L., and S. Siegel, *Bargaining Behavior*. New York: McGraw-Hill Book Co., 1963.

Grether, David M., "Bayes Rule as a Descriptive Model: The Representiveness Heuristic." *Quarterly Journal of Economics* 95(November):537–57, 1980.

Marschak, Jacob, "Economics of Inquiring, Communicating, Deciding." *American Economic Review* 58(May):1–18, 1968.

Shubik, Martin, "Oligopoly Theory, Communication, and Information." *American Economic Review* 65(May):280–83, 1975.

Siegel, Sydney, "Decision Making and Learning Under Varying Conditions of Reinforcement." *Annals of the New York Academy of Science* 89:766–83, 1961.

Smith, Vernon L., "Experimental Economics: Induced Value Theory." *American Economic Review* 66(May):274–79, 1976.

Vickrey, William, "Counterspeculation, Auctions and Competitive Sealed Tenders." *Journal of Finance* 16(May):8–37, 1961.